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ON THE STRONG SOLVABILITY OF PRIMITIVE EQUATIONS FOR THE COUPLED ATMOSPHERE AND OCEAN MODEL

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Annotation. This communication concerns the free boundary problem of primitive equations for the coupled atmosphere and ocean in three-dimensional strip with surface tension. By using the so-called p-coordinates and another coordinate transformation fixing the time-dependent domain, the temporally local existence of the unique strong solution to the transformed problem is proved in Sobolev–Slobodetskiĭ spaces.

Keywords: primitive equations, coupled atmosphere and ocean model, free boundary problem, Sobolev–Slobodetskiĭ spaces, strong solution

1. Introduction

The idea of numerical weather forecast was propounded by Richardson in the 1920 [1]. He derived some system of equations describing the motion of atmosphere (primitive equations for the atmosphere) that is similar to compressible Navier-Stokes equations. His attempt failed because of the lack of stability of calculations, notwithstanding however many attempts were carried out.

In 1969 Bryan [2] formulated the model of the ocean circulation (primitive equations for the ocean) similar to the Richardson model of the atmosphere by applying the hydrostatic approximation. In this model the Boussinesq approximation and the rigid lid hypothesis were used. The latter means that the ocean surface is fixed and flat. Then Crowley in [3] studied the free surface case rather than the rigid lid, for the ocean numerically.

One of the main features of primitive equations is the fact that the vertical velocity is determined by the horizontal velocities via the continuity equation, since the vertical velocity does not appear in the vertical component of equations of motion due to the hydrostatic approximation. For more details, see [4–7].

Mathematical studies of primitive equations began in the 1990. In [8, 9], Lions, Temam, and Wang studied an evolutionary 3D ocean model with rigid lid. Further, they formulated the coupled atmosphere and ocean model in [10, 11] by assuming that the height of the pressure isobar coincides with the height of the rigid lid. In [11, 12] they studied the model of [13] numerically and mathematically in the weak framework, respectively.

All papers cited above were discussed under the rigid lid hypothesis for primitive equations with the turbulent viscosity terms added later as an empirical claim. Moreover, the model equations were described in Cartesian coordinates for the ocean, while in p-coordinates for the atmosphere. These might lead to a discord of the interface between atmosphere and ocean, which causes a serious difficulty. To avoid such a situation, we used p-coordinates for the ocean model in [14] (see also [15]). In addition to the turbulent viscosity we take into account the effect of the surface movement following Crowley [3], and adopt f-plane approximation, i.e., Coriolis parameter is a constant, where f is the deformation angle from the sphere over the plane.

Here, we stress the major features of the present paper:

1) the free boundary problem for the coupled atmosphere and ocean is studied; i.e., the interface between the atmosphere and ocean is free rather than rigid lid approximated;

2) the boundary conditions on the interface are described by the stress tensor and the effect of vaporization;

3) the original equations are transformed by both the p-coordinates and the coordinate transformation from the time-dependent domain to some time-independent fixed domain;

4) a proof for the existence of a strong solution is given in Sobolev–Slobodetskiĭ spaces.

The rest of this paper is organized as follows. In Section 2, we give the mathematical formulation of our problem. Then the original problem is rewritten in p-coordinates, and another mapping is introduced to transform the problem into that on the time-independent fixed domain. In Section 3, we describe our main theorem in this paper. We use the anisotropic Sobolev–Slobodetskiĭ spaces $W_2^{l,l/2}(Q_T)$ $(Q_T \equiv \Omega \times (0,T))$ as solution spaces.

After that the nonhomogeneous linear problem on the fixed domain and then the nonlinear problem on a short time interval via the iteration method will be solved. But they all would be omitted here because of the limited spaces. The complete proof [16] will be published elsewhere.

2. Formulation of the Problem

In this section, we begin with formulating the problem of Section 1, which is the free boundary problem of primitive equations for the coupled atmosphere and ocean model.

By adopting f-plane approximation, our problem can be considered in the strip-like region in $\mathbf{R}^3$ with an orthogonal coordinate system $x = (x_1, x_2, x_3)$ in the vertical direction x_3. Suppose that the upper surface and the fixed bottom of the ocean are described by $x_3 = d(x', t)$ and $x_3 = b(x')$ $(x' = (x_1, x_2))$, respectively, where $b(x')$ is a given function. Let the oceanic region be denoted by

$$\Omega^s(t) = \{(x', x_3) \in \mathbf{R}^3 \mid b(x') < x_3 < d(x', t),\ x' \in \mathbf{R}^2\},$$

and the atmospheric region by

$$\Omega^a(t) = \{(x', x_3) \in \mathbf{R}^3 \mid d(x', t) < x_3 < H_0,\ x' \in \mathbf{R}^2\},$$

where H_0 is a constant representing a scale height. The equations under consideration in the present paper are as follows:

$$
\begin{gathered}
\frac{\partial \mathbf{v}^a}{\partial t} + (\mathbf{v}^a \cdot \nabla)\mathbf{v}^a + w^a \frac{\partial \mathbf{v}^a}{\partial x_3} - \frac{1}{\varrho^a}\left(\mu_1^a \triangle \mathbf{v}^a + \mu_2^a \frac{\partial^2 \mathbf{v}^a}{\partial x_3^2}\right) + f\mathbf{A}\mathbf{v}^a = -\frac{1}{\varrho^a}\nabla p^a + \mathbf{F}_1^a, \\
\frac{\partial p^a}{\partial x_3} = -\varrho^a g, \quad p^a = R\varrho^a\theta^a, \\
\frac{\partial \theta^a}{\partial t} + (\mathbf{v}^a \cdot \nabla)\theta^a + w^a \frac{\partial \theta^a}{\partial x_3} - \frac{1}{\varrho^a}\left(\mu_3^a \triangle \theta^a + \mu_4^a \frac{\partial^2 \theta^a}{\partial x_3^2}\right) = F_2^a, \\
\frac{\partial q}{\partial t} + (\mathbf{v}^a \cdot \nabla)q + w^a \frac{\partial q}{\partial x_3} - \frac{1}{\varrho^a}\left(\mu_5^a \triangle q + \mu_6^a \frac{\partial^2 q}{\partial x_3^2}\right) = F_3^a, \\
\frac{\partial \varrho^a}{\partial t} + \nabla \cdot (\varrho^a \mathbf{v}^a) + \frac{\partial}{\partial x_3}(\varrho^a w^a) = 0, \quad x \in \Omega^a(t),\ t > 0,
\end{gathered}
\tag{2.1}
$$

$$
\begin{gathered}
\frac{\partial \mathbf{v}^s}{\partial t} + (\mathbf{v}^s \cdot \nabla)\mathbf{v}^s + w^s \frac{\partial \mathbf{v}^s}{\partial x_3} - \frac{1}{\bar{\varrho}^s}\left(\mu_1^s \Delta \mathbf{v}^s + \mu_2^s \frac{\partial^2 \mathbf{v}^s}{\partial x_3^2}\right) + f\mathbf{A}\mathbf{v}^s = -\frac{1}{\bar{\varrho}^s}\nabla p^s + \mathbf{F}_1^s, \\
\frac{\partial p^s}{\partial x_3} = -\varrho^s g, \quad \varrho^s = \bar{\varrho}^s[1 - \beta_\theta(\theta^s - \bar{\theta}^s) + \beta_S(S - \bar{S})], \\
\frac{\partial \theta^s}{\partial t} + (\mathbf{v}^s \cdot \nabla)\theta^s + w^s \frac{\partial \theta^s}{\partial x_3} - \frac{1}{\bar{\varrho}^s}\left(\mu_3^s \Delta \theta^s + \mu_4^s \frac{\partial^2 \theta^s}{\partial x_3^2}\right) = F_2^s, \\
\frac{\partial S}{\partial t} + (\mathbf{v}^s \cdot \nabla)S + w^s \frac{\partial S}{\partial x_3} - \frac{1}{\bar{\varrho}^s}\left(\mu_5^s \Delta S + \mu_6^s \frac{\partial^2 S}{\partial x_3^2}\right) = F_3^s, \\
\nabla \cdot \mathbf{v}^s + \frac{\partial w^s}{\partial x_3} = 0, \quad x \in \Omega^s(t),\ t > 0.
\end{gathered}
\tag{2.2}
$$

Here $f\mathbf{A}\mathbf{v}$ is a Coriolis force with $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and Coriolis parameter f; ∇ and Δ are the 2-dimensional gradient and the Laplacian, respectively; $\mathbf{F}_1^\tau$ ($\tau = a, s$) are the horizontal component of external forces; $\mathbf{V}^\tau = (\mathbf{v}^\tau, w^\tau)$ ($\tau = a, s$) are the 3-dimensional velocity vector fields with a horizontal component $\mathbf{v}^\tau$ and a vertical component w^τ; while p^τ, ϱ^τ, and θ^τ ($\tau = a, s$) are the pressure, the density and the temperature, respectively; S is the salinity and q is the moisture; F_2^τ and F_3^τ ($\tau = a, s$) are the sources of heat and moisture ($\tau = a$) or salinity ($\tau = s$), respectively; R is the gas constant for the dry air [7]; μ_1^τ and μ_2^τ ($\tau = a, s$) are the coefficients of turbulent viscosity; (μ_3^τ, μ_4^τ) and (μ_5^τ, μ_6^τ) ($\tau = a, s$) are, respectively, given by scaling sum of turbulent and molecular diffusivities of heat and moisture ($\tau = a$) or salinity ($\tau = s$); $(\bar{\varrho}^s, \bar{\theta}^s, \bar{S})$ are the average values (constants) of density, temperature and salinity, respectively; and (β_θ, β_S) are the coefficients of expansion [8].

The conditions on the free surface $\Gamma(t) = \{(x', d(x', t)) \mid x' \in \mathbf{R}^2\}$, $t > 0$, are as follows:

$$
\mathbf{v}^a = \mathbf{v}^s,
$$

$$
\begin{aligned}
&[\mathbf{T}^a(\mathbf{v}^a) - \mathbf{T}^s(\mathbf{v}^s)]\mathbf{n} - ([\mathbf{T}^a(\mathbf{v}^a) - \mathbf{T}^s(\mathbf{v}^s)]\mathbf{n} \cdot \mathbf{n}')\mathbf{n}' \\
&\quad = |\mathbf{v}^a - \delta\mathbf{v}^s|^\alpha((\mathbf{v}^a - \delta\mathbf{v}^s) - ((\mathbf{v}^a - \delta\mathbf{v}^s) \cdot \mathbf{n}')\mathbf{n}'),
\end{aligned}
\tag{2.3}
$$

$$
\begin{gathered}
-\Big\{\mu_3^a\nabla\theta^a\cdot\mathbf{n}'+\mu_4^a\frac{\partial\theta^a}{\partial x_3}n_3\Big\}+\Big\{\mu_3^s\nabla\theta^s\cdot\mathbf{n}'+\mu_4^s\frac{\partial\theta^s}{\partial x_3}n_3\Big\}\\
=-la(\theta_e)\mathscr{V}+g_1|\mathbf{v}^a-\delta\mathbf{v}^s|^\alpha\theta_e+\sigma LH,\\
\theta^a=\theta^s=\theta_e,\quad p^a=p^s=p_0,\quad (q,S)=(q_e,S_e),\quad x\in\Gamma(t),\quad t>0,
\end{gathered}
$$

where

$$
\mathbf{T}^\tau(\mathbf{v}^\tau)=\begin{pmatrix}\mu_1^\tau\frac{\partial v_1^\tau}{\partial x_1} & \mu_1^\tau\frac{\partial v_1^\tau}{\partial x_2} & \mu_2^\tau\frac{\partial v_1^\tau}{\partial x_3}\\ \mu_1^\tau\frac{\partial v_2^\tau}{\partial x_1} & \mu_1^\tau\frac{\partial v_2^\tau}{\partial x_2} & \mu_2^\tau\frac{\partial v_2^\tau}{\partial x_3}\end{pmatrix}\quad(\tau=a,s); \tag{2.4}
$$

$\mathscr{V}$, the normal velocity of the free surface; $la(\theta_e)$, a latent heat with saturation temperature θ_e; σ, the surface tension coefficient; H, the twice mean curvature; $\mathbf{n}=(\mathbf{n}',n_3)=(n_1,n_2,n_3)$, the unit normal vector to $\Gamma(t)$ at time t pointing to the atmospheric region; L, the heat capacity; g_1, a given function representing the turbulent transition on the free surface including the albedo of the earth; p_0, a given pressure on the free surface assumed to be a positive constant; δ, the ratio between the typical velocity of the ocean and that of the atmosphere.

The boundary conditions on the bottom of the ocean $\Gamma^{(s)}=\{(x',b(x'))\mid x'\in\mathbf{R}^2\}$ and the upper surface of the atmosphere $\Gamma^{(a)}=\{(x',H_0)\mid x'\in\mathbf{R}^2\}$ are

$$
\begin{gathered}
(\mathbf{V}^s,\theta^s,S)(x,t)=(\mathbf{0},\theta_b,S_b)(x,t),\quad x\in\Gamma^{(s)},\ t>0,\\
\frac{\partial\mathbf{v}^a}{\partial x_3}=\alpha_v(\mathbf{v}_c-\mathbf{v}^a),\quad w^a=0,\\
\frac{\partial\theta^a}{\partial x_3}=\alpha_\theta(\theta_c-\theta^a),\quad \frac{\partial q}{\partial x_3}=\alpha_q(q_c-q),\quad x\in\Gamma^{(a)},\ t>0,
\end{gathered} \tag{2.5}
$$

where α_v, α_θ, and α_q are positive constants, and $\mathbf{v}_c$, θ_c and q_c are horizontal velocity, temperature and moisture of the layer above the scale height, respectively.

The initial conditions are

$$
\begin{gathered}
(\mathbf{v}^a,\theta^a,q)(x,0)=\big(\mathbf{v}_0^a,\theta_0^a,q_0\big)(x),\quad x\in\Omega^a\equiv\Omega^a(0),\\
(\mathbf{v}^s,\theta^s,S)(x,0)=\big(\mathbf{v}_0^s,\theta_0^s,S_0\big)(x),\quad x\in\Omega^s\equiv\Omega^s(0),\\
d(x',0)=d_0(x'),\quad x'\in\mathbf{R}^2.
\end{gathered} \tag{2.6}
$$

Now, we introduce the *p-coordinate system.* From $(2.1)_2$, $(2.2)_2$ and $(2.3)_5$ p^a and p^s can be represented as

$$
p^a(x,t)=p_0\exp\left(-\int_{d(x',t)}^{x_3}\frac{g}{R\theta^a(x',z,t)}\,\mathrm{d}z\right), \tag{2.7}
$$

$$
\begin{aligned}
p^s(x,t)=p_0-(x_3-d(x',t))\bar\varrho^s g+\bar\varrho^s g\beta_\theta\int_{d(x',t)}^{x_3}(\theta^s-\bar\theta^s)(x',z,t)\,\mathrm{d}z\\
-\bar\varrho^s g\beta_S\int_{d(x',t)}^{x_3}(S-\bar S)(x',z,t)\,\mathrm{d}z.
\end{aligned}
$$

Since $\partial p^\tau/\partial x_3<0$ $(\tau=a,s)$, we can take the pressures at the upper surface of the atmosphere and the bottom of the ocean as independent variables in place of x_3.

Put

$$h^a(x',t) = p_0 \exp\left(-\int\limits_{d(x',t)}^{H_0} \frac{g}{R\theta^a(x',z,t)}\,\mathrm{d}z \right),$$

$$h^s(x',t) = p_0 - (b(x') - d(x',t))\bar{\varrho}^s g + \bar{\varrho}^s g\beta_\theta \int\limits_{d(x',t)}^{b(x')} (\theta^s - \bar{\theta}^s)(x',z,t)\,\mathrm{d}z$$

$$-\bar{\varrho}^s g\beta_S \int\limits_{d(x',t)}^{b(x')} (S - \bar{S})(x',z,t)\,\mathrm{d}z. \tag{2.8}$$

Obviously $h^\tau(x',t)$ $(\tau = a, s)$ are unknown since $d(x',t)$ is unknown. Set

$$h_0^a(x') = h^a(x',0), \quad h_0^s(x') = h^s(x',0). \tag{2.9}$$

Here we assume that $H_0 > d_0(x') > b(x')$ holds for any $x' \in \mathbf{R}^2$. Then there exist inverse functions $d(x',t) = \Psi^a(x',t;\theta^a,h^a)$ and $x_3 = X_3^a(x',p^a,t;\theta^a,h^a)$ of $(2.8)_1$ and $(2.7)_1$ for the atmosphere, and $d(x',t) = \Psi^s(x',t;\theta^s,S,h^s)$ and $x_3 = X_3^s(x',p^s,t;\theta^s,S,h^s)$ of $(2.8)_2$ and $(2.7)_2$ for the ocean, respectively. From (2.7) it is easy to derive that

$$\nabla p^a = p^a \left(\frac{g}{R\theta^a}\bigg|_{x_3=d(x',t)} \nabla d + \frac{g}{R} \int\limits_{d(x',t)}^{x_3} \frac{\nabla\theta^a}{{\theta^a}^2}(x',z,t)\,\mathrm{d}z \right) \equiv \mathbf{F}_5^a(x,t),$$

$$\frac{\partial p^a}{\partial t} = p^a \left(\frac{g}{R\theta^a}\bigg|_{x_3=d(x',t)} \frac{\partial d}{\partial t} + \frac{g}{R} \int\limits_{d(x',t)}^{x_3} \frac{1}{{\theta^a}^2}\frac{\partial\theta^a}{\partial t}(x',z,t)\,\mathrm{d}z \right) \equiv F_6^a(x,t),$$

$$\nabla p^s = \bar{\varrho}^s g \nabla d (1 - \beta_\theta(\theta^s - \bar{\theta}^s) + \beta_S(S - \bar{S}))|_{x_3=d(x',t)}$$
$$+\bar{\varrho}^s g \int\limits_{d(x',t)}^{x_3} (\beta_\theta \nabla\theta^s(x',z,t) - \beta_S \nabla S(x',z,t))\,\mathrm{d}z \equiv \mathbf{F}_5^s(x,t),$$

$$\frac{\partial p^s}{\partial t} = \bar{\varrho}^s g \frac{\partial d}{\partial t}(1 - \beta_\theta(\theta^s - \bar{\theta}^s) + \beta_S(S - \bar{S}))|_{x_3=d(x',t)}$$
$$+\bar{\varrho}^s g \int\limits_{d(x',t)}^{x_3} \left(\beta_\theta \frac{\partial\theta^s}{\partial t}(x',z,t) - \beta_S \frac{\partial S}{\partial t}(x',z,t) \right) \mathrm{d}z \equiv F_6^s(x,t). \tag{2.10}$$

Note that after introducing p-coordinates, the interface between the atmosphere and the ocean becomes flat, represented by $p^a = p^s = p_0$, while the upper surface of the atmosphere given by $p^a = h^a(x',t)$ and the bottom of the ocean given by $p^s = h^s(x',t)$ are unknown.

Hereafter, in order to indicate the dependence on $\theta^{\tau *} = \theta^\tau \circ X_3^\tau$, $h^\tau, \ldots$ $(\tau = a, s)$ explicitly after this transformation, we use the following notation: for a function $F(x', x_3, t)$ on $\Omega^a(t)$

$$F^{(\theta^{a*}, h^a)}(x', p^a, t) \equiv (F \circ X_3^a)(x', p^a, t) = F(x', X_3^a(x', p^a, t; \theta^a, h^a), t);$$

for F on $\Omega^s(t)$

$$F^{(\theta^{s*}, S^*, h^s)}(x', p^s, t) \equiv (F \circ X_3^s)(x', p^s, t) = F(x', X_3^s(x', p^s, t; \theta^s, S, h^s), t).$$

In addition, for the atmosphere, (2.10) implies

$$\bar{w}^a \equiv \frac{\mathrm{d}p^a}{\mathrm{d}t} \equiv \frac{\partial p^a}{\partial t} + (\mathbf{v}^a \cdot \nabla) p^a + w^a \frac{\partial p^a}{\partial x_3} = F_6^a + \mathbf{v}^a \cdot \mathbf{F}_5^a + w^a \frac{\partial p^a}{\partial x_3}$$

(cf. [8, 15]), and hence

$$\nabla \cdot \mathbf{v}^{a*} + \frac{\partial \bar{w}^{a*}}{\partial p^a} = 0.$$

Now we introduce the mappings $(x', p^\tau) \longmapsto (y', y_3)$ $(\tau = a, s)$:

$$y' = x', \quad y_3 = \big(p_0 - h_0^\tau(x')\big) \frac{p^\tau - h^\tau(x', t)}{p_0 - h^\tau(x', t)} + h_0^\tau(x') \tag{2.11}$$

which are similar to those in [17]. By composing the p-coordinates and (2.11), it is clear that the regions

$$\bigcup_{0<t<T} (\Omega^\tau(t) \times \{t\}), \quad \bigcup_{0<t<T} (\Gamma^{(\tau)} \times \{t\}) \ (\tau = a, s), \quad \bigcup_{0<t<T} (\Gamma(t) \times \{t\})$$

are transformed onto the regions

$$\widetilde{\Omega}_T^\tau \equiv \widetilde{\Omega}^\tau \times (0, T), \quad \widetilde{\Gamma}_T^{(\tau)} \equiv \widetilde{\Gamma}^{(\tau)} \times (0, T) \ (\tau = a, s), \quad \widetilde{\Gamma}_T \equiv \widetilde{\Gamma} \times (0, T),$$

respectively, where

$$\widetilde{\Omega}^a = \big\{(y', y_3) \in \mathbf{R}^3 \mid y' \in \mathbf{R}^2,\ h_0^a(y') < y_3 < p_0\big\},$$
$$\widetilde{\Omega}^s = \big\{(y', y_3) \in \mathbf{R}^3 \mid y' \in \mathbf{R}^2,\ p_0 < y_3 < h_0^s(y')\big\},$$
$$\widetilde{\Gamma}^{(\tau)} = \big\{(y', y_3) \mid y' \in \mathbf{R}^2,\ y_3 = h_0^\tau(y')\big\} \quad (\tau = a, s),$$
$$\widetilde{\Gamma} = \{(y', y_3) \mid y' \in \mathbf{R}^2,\ y_3 = p_0\}.$$

Let $\mathscr{A}^\tau$ $(\tau = a, s)$ be the inverse of the transposed matrices of the Jacobian matrices of the transformations (2.11) by

$$\mathscr{A}^\tau = (a_\tau^{ij}) = (a_\tau^{ij}(h^\tau)) \quad (i, j = 1, 2, 3;\ \tau = a, s),$$

whose elements are given as

$$\mathbf{a}_\tau^3 = \mathbf{a}_\tau^3(h^\tau) \equiv \big(a_\tau^{31}, a_\tau^{32}\big)^{\mathrm{T}} = \frac{(p_0 - h_0^\tau)(\tilde{p}^\tau(y, t) - p_0)}{(p_0 - h^\tau)^2} \nabla h^\tau + \frac{p_0 - \tilde{p}^\tau(y, t)}{p_0 - h^\tau} \nabla h_0^\tau$$
$$\equiv A_1^\tau(y, t) \nabla h^\tau + \mathbf{B}_1^\tau(y, t),$$

$$p^\tau(x, t) = \tilde{p}^\tau(y, t) \equiv \frac{(p_0 - h^\tau)(y_3 - h_0^\tau)}{p_0 - h_0^\tau} + h^\tau,$$

$$a_\tau^{33} = \frac{p_0 - h_0^\tau}{p_0 - h^\tau}, \quad a_\tau^{ij} = \delta_{ij} \quad (i = 1, 2,\ j = 1, 2, 3; \tau = a, s).$$

In what follows, for brevity we use the notation

$$\begin{pmatrix} \nabla_{h^\tau} \\ \nabla_{h^\tau,3} \end{pmatrix} = \mathscr{A} \begin{pmatrix} \nabla \\ \frac{\partial}{\partial p^\tau} \end{pmatrix},$$

$$\theta^{\tau**}(y', y_3, t) = \theta^{\tau*}(x', p^\tau, t) \quad (\tau = a, s),$$

$$\tilde{X}_3^a(y', y_3, t; \theta^{a**}, h^a) = X_3^a(x', p^a, t; \theta^{a*}, h^a)\big|_{x'=y', p^a=\tilde{p}^a(y,t)},$$

$$\tilde{X}_3^s(y', y_3, t; \theta^{s**}, S^{**}, h^s) = X_3^s(x', p^s, t; \theta^{s*}, S^*, h^s)|_{x'=y', p^s=\tilde{p}^s(y,t)},$$

$$f^{(\theta^{a**}, h^a)}(y', y_3, t) = f^{(\theta^{a*}, h^a)}(x', p^a, t)|_{x'=y', p^a=\tilde{p}^a(y,t)},$$

$$f^{(\theta^{s**}, S^{**}, h^a)}(y', y_3, t) = f^{(\theta^{s*}, S^*, h^a)}(x', p^s, t)|_{x'=y', p^s=\tilde{p}^s(y,t)}.$$

The above coordinate transformations turn $\mathbf{T}^\tau \mathbf{n}$ ($\tau = a, s$) on $\Gamma(t)$ into

$$\widetilde{\mathbf{T}}^a_{\theta^{a**}, h^a}(\mathbf{v}^{a**}) \equiv \mu_1^a \left(\mathbf{n}^{a\prime} \cdot \left(\nabla_{h^a} + \mathbf{F}_5^{a(\theta^{a**}, h^a)} a_a^{33} \frac{\partial}{\partial y_3} \right) \right) \mathbf{v}^{a**} \\ - \mu_2^a n_3 \frac{\tilde{p}^a g}{R\theta^{a**}} a_a^{33} \frac{\partial \mathbf{v}^{a**}}{\partial y_3} \bigg|_{y_3=p_0},$$

$$\widetilde{\mathbf{T}}^s_{\theta^{s**}, S^{**}, h^s}(\mathbf{v}^{s**}) \equiv \mu_1^s \left(\mathbf{n}^{s\prime} \cdot \left(\nabla_{h^s} + \mathbf{F}_5^{s(\theta^{s**}, S^{**}, h^s)} a_s^{33} \frac{\partial}{\partial y_3} \right) \right) \mathbf{v}^{s**} \\ - \mu_2^s n_3 \varrho^{s**} g a_s^{33} \frac{\partial \mathbf{v}^{s**}}{\partial y_3} \bigg|_{y_3=p_0}$$

with

$$\mathbf{n}^a = \left(\mathbf{n}^{a\prime}, n_3^a\right)^{\mathrm{T}} = \frac{\left(\mathbf{a}_a^3 + a_a^{33} \mathbf{F}_5^{a(\theta^{a**}, h^a)}, -\frac{\tilde{p}^a g}{R\theta^{a**}} a_a^{33}\right)^{\mathrm{T}}}{\left|\left(\mathbf{a}_a^3 + a_a^{33} \mathbf{F}_5^{a(\theta^{a**}, h^a)}, -\frac{\tilde{p}^a g}{R\theta^{a**}} a_a^{33}\right)\right|}\Bigg|_{y_3=p_0},$$

$$\mathbf{n}^s = \left(\mathbf{n}^{s\prime}, n_3^s\right)^{\mathrm{T}} = \frac{\left(\mathbf{a}_s^3 + a_s^{33} \mathbf{F}_5^{s(\theta^{s**}, S^{**}, h^s)}, -\varrho^{s**} g a_s^{33}\right)^{\mathrm{T}}}{\left|\left(\mathbf{a}_s^3 + a_s^{33} \mathbf{F}_5^{s(\theta^{s**}, S^{**}, h^s)}, -\varrho^{s**} g a_s^{33}\right)\right|}\Bigg|_{y_3=p_0},$$

$$\tilde{\varrho}^s = \bar{\varrho}^s[1 - \beta_\theta(\tilde{\theta}^s - \bar{\theta}^s) + \beta_S(\widetilde{S} - \bar{S})] \equiv \varrho^{s**}.$$

Now, the problem (2.1)–(2.6) is rewritten in y-coordinates for the unknowns

$$\left(\mathbf{u}^a, u_3^a, \tilde{\theta}^a, \tilde{q}\right) \equiv (\mathbf{v}^{a**}, \bar{w}^{a**}, \theta^{a**}, q^{**}),$$

$$\left(\mathbf{u}^s, u_3^s, \tilde{\theta}^s, \widetilde{S}\right) \equiv (\mathbf{v}^{s**}, w^{s**}, \theta^{s**}, S^{**}), \quad h^a, \quad h^s,$$

where

$$(\mathbf{v}^{a**}, \bar{w}^{a**}, \theta^{a**}, q^{**})(y', y_3, t) = (\mathbf{v}^{a*}, \bar{w}^{a*}, \theta^{a*}, q^*)(x', p^a, t) \\ = (\mathbf{v}^a, \bar{w}^a, \theta^a, q)(x', x_3, t),$$

$$(\mathbf{v}^{s**}, w^{s**}, \theta^{s**}, S^{**})(y', y_3, t) = (\mathbf{v}^{s*}, w^{s*}, \theta^{s*}, S^*)(x', p^s, t) \\ = (\mathbf{v}^s, w^s, \theta^s, S)(x', x_3, t),$$

$$(h^a, h^s)(y', t) = (h^a, h^s)(x', t).$$

For brevity, we introduce the notation

$$\mathscr{U}^a \equiv (\mathbf{u}^a, \tilde{\theta}^a, \tilde{q}), \quad \mathscr{U}^s \equiv (\mathbf{u}^s, \tilde{\theta}^s, \widetilde{S}).$$

Then

$$\begin{aligned}
&\frac{\partial \mathscr{U}^a}{\partial t} - \mathscr{L}^{(a)}_{\mathscr{U}^a, h^a} \mathscr{U}^a = \mathbf{l}^a\big(\mathscr{U}^a, u_3^a, h^a\big) \quad \text{in } \widetilde{\Omega}_T^a,\\
&\frac{\partial \mathscr{U}^s}{\partial t} - \mathscr{L}^{(s)}_{\mathscr{U}^s, h^s} \mathscr{U}^s = \mathbf{l}^s\big(\mathscr{U}^s, u_3^s, h^s\big) \quad \text{in } \widetilde{\Omega}_T^s,\\
&\mathscr{B}_{\mathscr{U}^a, \mathscr{U}^s, h^a, h^s}(\mathscr{U}^a, \mathscr{U}^s) = \mathbf{b}_1(\mathscr{U}^a, \mathscr{U}^s, h^a, h^s) \quad \text{on } \widetilde{\Gamma}_T,\\
&\frac{\partial \mathscr{U}^a}{\partial y_3} = \mathbf{b}_2(\mathscr{U}^a, h^a) \quad \text{on } \widetilde{\Gamma}_T^{(a)},\\
&\mathscr{U}^s = \mathbf{b}_3 \quad \text{on } \widetilde{\Gamma}_T^{(s)}, \quad \mathscr{U}^\tau|_{t=0} = \mathscr{U}_0^\tau \quad \text{on } \widetilde{\Omega}^\tau \ (\tau = a, s),
\end{aligned} \tag{2.12}$$

$$\begin{aligned}
&-\Big\{\mu_3^a\Big(\nabla_{h^a} + a_a^{33}\mathbf{F}_5^{a(\tilde{\theta}^a, h^a)}\frac{\partial}{\partial y_3}\Big)\tilde{\theta}^a \cdot \mathbf{n}^{a\prime} - \mu_4^a a_a^{33}\frac{g p_0}{R\tilde{\theta}^a}\frac{\partial \tilde{\theta}^a}{\partial y_3} n_3^a\Big\}\\
&+\Big\{\mu_3^s\Big(\nabla_{h^s} + a_s^{33}\mathbf{F}_5^{s(\tilde{\theta}^s, \widetilde{S}^s, h^s)}\frac{\partial}{\partial y_3}\Big)\tilde{\theta}^s \cdot \mathbf{n}^{s\prime} - \mu_4^a a_a^{33}\tilde{\varrho}^s g\frac{\partial \tilde{\theta}^a}{\partial y_3} n_3^s\Big\}\\
&= -la\big(\theta_e^{(\tau)}\big)\mathscr{V}^{(\tau)} + g_1^{(\tau)}|\mathbf{u}^a - \delta\mathbf{u}^s|^\alpha \theta_e^{(\tau)} + \sigma L H^{(\tau)} \quad \text{on } \widetilde{\Gamma}_T \quad (\tau = a, s),\\
&\nabla_{h^\tau, 3}\, u_3^\tau = G^\tau_{3, \mathscr{U}^\tau, h^\tau}(\mathbf{u}^\tau) \quad \text{in } \widetilde{\Omega}_T^{(\tau)}, \quad u_3^\tau = 0 \quad \text{on } \widetilde{\Gamma}_T^{(\tau)} \ (\tau = a\ s),
\end{aligned} \tag{2.13}$$

where

$$\begin{aligned}
&\mathscr{L}^{(a)}_{\mathscr{U}^a, h^a}\mathscr{U}^a = \big(L^a_{1, \tilde{\theta}^a, h^a}\mathbf{u}^a, L^a_{2, \tilde{\theta}^a, h^a}\tilde{\theta}^a, L^a_{3, \tilde{\theta}^a, h^a}\tilde{q}\big),\\
&\mathscr{L}^{(s)}_{\mathscr{U}^s, h^s}\mathscr{U}^s = \big(L^s_{1, \tilde{\theta}^s, \widetilde{S}^s, h^s}\mathbf{u}^s, L^s_{2, \tilde{\theta}^s, \widetilde{S}^s, h^s}\tilde{\theta}^s, L^s_{3, \tilde{\theta}^s, \widetilde{S}^s, h^s}\widetilde{S}\big),\\
&\mathbf{l}^a\big(\mathscr{U}^a, u_3^a, h^a\big) \equiv \big(\mathbf{G}^a_{1, \tilde{\theta}^a, h^a}\big(\mathbf{u}^a, u_3^a, \tilde{\theta}^a\big), G^a_{4, \tilde{\theta}^a, h^a}\big(\mathbf{u}^a, u_3^a, \tilde{\theta}^a\big), G^a_{5, \tilde{\theta}^a, h^a}\big(\mathbf{u}^a, u_3^a, \tilde{q}\big)\big),\\
&\mathbf{l}^s\big(\mathscr{U}^s, u_3^s, h^s\big) \equiv \big(\mathbf{G}^s_{1, \tilde{\theta}^s, \widetilde{S}^s, h^s}\big(\mathbf{u}^s, u_3^s\big), G^s_{4, \tilde{\theta}^s, \widetilde{S}^s, h^s}\big(\mathbf{u}^s, u_3^s, \tilde{\theta}^s\big), G^s_{5, \tilde{\theta}^s, \widetilde{S}^s, h^s}\big(\mathbf{u}^s, u_3^s, \widetilde{S}\big)\big),\\
&\mathscr{B}_{\mathscr{U}^a, \mathscr{U}^s, h^a, h^s}(\mathscr{U}^a, \mathscr{U}^s) = (\mathbf{u}^a - \mathbf{u}^s, \mathbf{B}_{\tilde{\theta}^a, h^a, \tilde{\theta}^s, \widetilde{S}, h^s}(\mathbf{u}^a, \mathbf{u}^s), \tilde{\theta}^a, \tilde{q}, \tilde{\theta}^s, \widetilde{S}),
\end{aligned}$$

$$\begin{aligned}
\mathbf{B}_{\tilde{\theta}^a, h^a, \tilde{\theta}^s, \widetilde{S}, h^s}(\mathbf{u}^a, \mathbf{u}^s) &= \widetilde{\mathbf{T}}^a_{\tilde{\theta}^a, h^a}(\mathbf{u}^a) - \widetilde{\mathbf{T}}^s_{\tilde{\theta}^s, \widetilde{S}, h^s}(\mathbf{u}^s)\\
&-\big[\big(\widetilde{\mathbf{T}}^a_{\tilde{\theta}^a, h^a}(\mathbf{u}^a) \cdot \mathbf{n}^{a\prime}\big)\mathbf{n}^{a\prime} - \big(\widetilde{\mathbf{T}}^s_{\tilde{\theta}^s, \widetilde{S}, h^s,}(\mathbf{u}^s) \cdot \mathbf{n}^{s\prime}\big)\mathbf{n}^{s\prime}\big],
\end{aligned}$$

$$\begin{aligned}
&\mathbf{b}_1(\mathscr{U}^a, \mathscr{U}^s, h^a, h^s) = \big(\mathbf{0}, \mathbf{G}_{2, \tilde{\theta}^a, h^a, \tilde{\theta}^s, \widetilde{S}, h^s}(\mathbf{u}^a, \mathbf{u}^s), \tilde{\theta}_e^a, \tilde{q}_e, \tilde{\theta}_e^s, \widetilde{S}_e\big),\\
&\mathbf{b}_2(\mathscr{U}^a, h^a) = -\frac{R\tilde{\theta}^a}{p_0 g a_a^{33}}(\alpha_v(\mathbf{v}_c - \mathbf{u}^a), \alpha_\theta(\theta_c - \tilde{\theta}^a), \alpha_c(q_c - \tilde{q})),\\
&\mathbf{b}_3 = (\mathbf{0}, \theta_b, S_b),\\
&\mathscr{U}_0^a = \big(\mathbf{v}_0^{a(\tilde{\theta}_0^a, h_0^a)}, \theta_0^{a(\tilde{\theta}_0^a, h_0^a)}, q_0^{(\tilde{\theta}_0^a, h_0^a)}\big), \quad \mathscr{U}_0^s = \big(\mathbf{v}_0^{s(\tilde{\theta}_0^s, \widetilde{S}_0, h_0^s)}, \theta_0^{s(\tilde{\theta}_0^s, \widetilde{S}_0, h_0^s)}, S_0^{(\tilde{\theta}_0^s, \widetilde{S}_0, h_0^s)}\big),\\
&\big(\tilde{\theta}_e^a, \tilde{q}_e\big) = (\theta_e, q_e)|_{y_3 = \Psi^a}, \quad \big(\tilde{\theta}_e^s, \widetilde{S}_e\big) = (\theta_e, S_e)|_{y_3 = \Psi^s},\\
&\tilde{\varrho}^s = \bar{\varrho}^s[1 - \beta_\theta(\tilde{\theta}^s - \bar{\theta}^s) + \beta_S(\widetilde{S} - \bar{S})] \equiv \varrho^{s**},\\
&L^a_{1, \tilde{\theta}^a, h^a} \equiv \mu_1^a L^a_{11, \tilde{\theta}^a, h^a} + \mu_2^a L^a_{12, \tilde{\theta}^a, h^a},
\end{aligned}$$

$$L^a_{11,\tilde{\theta}^a,h^a} = \frac{R\tilde{\theta}^a}{\tilde{p}^a}\Big(l^a_{11,h^a} + 2l^a_{12,\tilde{\theta}^a,h^a} + \big|\mathbf{F}_5^{a(\tilde{\theta}^a,h^a)}\big|^2\big(a_a^{33}\big)^2\frac{\partial^2}{\partial y_3^2}\Big),$$

$$L^a_{12,\tilde{\theta}^a,h^a} = \frac{g^2\tilde{p}^a}{R\tilde{\theta}^a}\big(a_a^{33}\big)^2\frac{\partial^2}{\partial y_3^2},$$

$$l^a_{11,h^a} = \nabla^2 + 2\mathbf{a}_a^3\cdot\nabla\frac{\partial}{\partial y_3} + \big|\mathbf{a}_a^3\big|^2\frac{\partial^2}{\partial y_3^2}, \quad l^a_{12,\tilde{\theta}^a,h^a} = a_a^{33}\mathbf{F}_5^{a(\tilde{\theta}^a,h^a)}\cdot\nabla_{h^a}\frac{\partial}{\partial y_3},$$

$$L^s_{1,\tilde{\theta}^s,\widetilde{S},h^s} \equiv \mu_1^s L^s_{11,\tilde{\theta}^s,\widetilde{S},h^s} + \mu_2^s L^s_{12,\tilde{\theta}^s,\widetilde{S},h^s},$$

$$L^s_{11,\tilde{\theta}^s,\widetilde{S},h^s} = \frac{1}{\tilde{\varrho}^s}\Big(l^s_{11,\tilde{\theta}^s,\widetilde{S},h^s} + 2l^s_{12,\tilde{\theta}^s,\widetilde{S},h^s} + \big|\mathbf{F}_5^{s(\tilde{\theta}^s,\widetilde{S},h^s)}\big|^2\big(a_s^{33}\big)^2\frac{\partial^2}{\partial y_3^2}\Big),$$

$$L^s_{12,\tilde{\theta}^s,\widetilde{S},h^s} = \frac{(\tilde{\varrho}^s g)^2}{\tilde{\varrho}^s}\big(a_s^{33}\big)^2\frac{\partial^2}{\partial y_3^2},$$

$$l^s_{11,\tilde{\theta}^s,\widetilde{S},h^s} = \nabla^2 + 2\mathbf{a}_s^3\cdot\nabla\frac{\partial}{\partial y_3} + \big|\mathbf{a}_s^3\big|^2\frac{\partial^2}{\partial y_3^2}, \quad l^s_{12,\tilde{\theta}^s,\widetilde{S},h^s} = a_s^{33}\mathbf{F}_5^{s(\tilde{\theta}^s,\widetilde{S},h^s)}\cdot\nabla_{h^s}\frac{\partial}{\partial y_3},$$

$$L^a_{i,\tilde{\theta}^a,h^a} \equiv \mu^a_{2i-1}L^a_{11,\tilde{\theta}^a,h^a} + \mu^a_{2i}L^a_{12,\tilde{\theta}^a,h^a},$$

$$L^s_{i,\tilde{\theta}^s,\widetilde{S},h^s} \equiv \mu^s_{2i-1}L^s_{11,\tilde{\theta}^s,\widetilde{S},h^s} + \mu^s_{2i}L^s_{12,\tilde{\theta}^s,\widetilde{S},h^s} \quad (i=2,3),$$

$$\begin{aligned}
\mathbf{G}^a_{1,\tilde{\theta}^a,h^a}\big(\mathbf{u}^a,u_3^a,\tilde{\theta}^a\big) &= \frac{\mu_1^a R\tilde{\theta}^a}{\tilde{p}^a}\Big(\nabla^2_{h^a} - l^a_{11,h^a} + \nabla_{h^a}\cdot\big(a_a^{33}\mathbf{F}_5^{a(\tilde{\theta}^a,h^a)}\big)\frac{\partial}{\partial y_3}\\
&\quad + a_a^{33}\mathbf{F}_5^{a(\tilde{\theta}^a,h^a)}\cdot\frac{\partial}{\partial y_3}\big(a_a^{33}\mathbf{F}_5^{a(\tilde{\theta}^a,h^a)}\big)\frac{\partial}{\partial y_3}\Big)\mathbf{u}^a\\
&\quad + \frac{\mu_2^a R\tilde{\theta}^a}{\tilde{p}^a}\Big(\big(a_a^{33}\frac{\tilde{p}^a g}{R\tilde{\theta}^a}\frac{\partial}{\partial y_3}\big)^2 - \big(a_a^{33}\frac{\tilde{p}^a g}{R\tilde{\theta}^a}\big)^2\frac{\partial^2}{\partial y_3^2}\Big)\mathbf{u}^a\\
&\quad -\Big((\mathbf{u}^a\cdot\nabla_{h^a}) + \big(\mathbf{F}_5^{a(\tilde{\theta}^a,h^a)}\cdot\mathbf{u}^a\big)a_a^{33}\frac{\partial}{\partial y_3} + a_a^{33}\big(u_3^a - F_6^{a(\tilde{\theta}^a,h^a)} - \mathbf{u}^a\cdot\mathbf{F}_5^{a(\tilde{\theta}^a,h^a)}\big)\frac{\partial}{\partial y_3}\\
&\quad + F_6^{a(\tilde{\theta}^a,h^a)}a_a^{33}\frac{\partial}{\partial y_3} + A_1^a\frac{\partial h^a}{\partial t}\frac{\partial}{\partial y_3}\Big)\mathbf{u}^a + f\mathbf{A}\mathbf{u}^a - \frac{R\tilde{\theta}^a}{\tilde{p}^a}\mathbf{F}_5^{a(\tilde{\theta}^a,h^a)} + \mathbf{F}_1^{a(\tilde{\theta}^a,h^a)}\\
&\equiv \mu_1^a G^a_{11,\tilde{\theta}^a,h^a}\mathbf{u}^a + \mu_2^a G^a_{12,\tilde{\theta}^a,h^a}\mathbf{u}^a - G^a_{13,\tilde{\theta}^a,h^a}(\mathbf{u}^a,u_3^a)\mathbf{u}^a\\
&\quad + f\mathbf{A}\mathbf{u}^a - \frac{R\tilde{\theta}^a}{\tilde{p}^a}\mathbf{F}_5^{a(\tilde{\theta}^a,h^a)} + \mathbf{F}_1^{a(\tilde{\theta}^a,h^a)},
\end{aligned}$$

$$\begin{aligned}
\mathbf{G}^s_{1,\tilde{\theta}^s,\widetilde{S},h^s}\big(\mathbf{u}^s,u_3^s\big) &= \frac{\mu_1^s}{\tilde{\varrho}^s}\Big(\nabla^2_{h^s} - l^s_{11,\tilde{\theta}^s,\widetilde{S},h^s} + \nabla_{h^s}\cdot\big(a_s^{33}\mathbf{F}_5^{s(\tilde{\theta}^s,\widetilde{S},h^s)}\big)\frac{\partial}{\partial y_3}\\
&\quad + a_s^{33}\mathbf{F}_5^{s(\tilde{\theta}^s,\widetilde{S},h^s)}\cdot\frac{\partial}{\partial y_3}\big(a_a^{33}\mathbf{F}_5^{s(\tilde{\theta}^s,\widetilde{S},h^s)}\big)\frac{\partial}{\partial y_3}\Big)\mathbf{u}^s\\
&\quad + \frac{\mu_2^s}{\tilde{\varrho}^s}\Big(\Big(a_s^{33}\tilde{\varrho}^s g\frac{\partial}{\partial y_3}\Big)^2 - \big(a_s^{33}\tilde{\varrho}^s g\big)^2\frac{\partial^2}{\partial y_3^2}\Big)\mathbf{u}^s - \Big((\mathbf{u}^s\cdot\nabla_{h^s}) + \big(\mathbf{F}_5^{s(\tilde{\theta}^s,\widetilde{S},h^s)}\cdot\mathbf{u}^s\big)a_s^{33}\frac{\partial}{\partial y_3}\\
&\quad + u_3^s a_s^{33}\frac{\partial}{\partial y_3} + F_6^{s(\tilde{\theta}^s,\widetilde{S},h^s)}a_s^{33}\frac{\partial}{\partial y_3} + A_1^s\frac{\partial h^s}{\partial t}\frac{\partial}{\partial y_3}\Big)\mathbf{u}^s\\
&\quad + f\mathbf{A}\mathbf{u}^s - \frac{1}{\tilde{\varrho}^s}\mathbf{F}_5^{s(\tilde{\theta}^s,\widetilde{S},h^s)} + \mathbf{F}_1^{s(\tilde{\theta}^s,\widetilde{S},h^s)}\\
&\equiv \mu_1^s G^s_{11,\tilde{\theta}^s,\widetilde{S},h^s}\mathbf{u}^s + \mu_2^s G^s_{12,\tilde{\theta}^s,\widetilde{S},h^s}\mathbf{u}^s - G^s_{13,\tilde{\theta}^s,\widetilde{S},h^s}(\mathbf{u}^s,u_3^s)\mathbf{u}^s + f\mathbf{A}\mathbf{u}^s\\
&\quad - \frac{1}{\tilde{\varrho}^s}\mathbf{F}_5^{s(\tilde{\theta}^s,\widetilde{S},h^s)} + \mathbf{F}_1^{s(\tilde{\theta}^s,\widetilde{S},h^s)},
\end{aligned}$$

$$G^a_{3,\mathscr{U}^a,h^a}(\mathbf{u}^a) = -\nabla_{h^a}\cdot\mathbf{u}^a - \mathbf{a}^3_a\cdot\frac{\partial\mathbf{u}^a}{\partial y_3},$$

$$G^s_{3,\mathscr{U}^s,h^s}(\mathbf{u}^s) = -\nabla_{h^s}\cdot\mathbf{u}^s - a^{33}_s\mathbf{F}_5^{s(\tilde\theta^s,\widetilde S,h^s)}\cdot\frac{\partial\mathbf{u}^s}{\partial y_3},$$

$$G^a_{4,\tilde\theta^a,h^a}(\mathbf{u}^a,u^a_3,\tilde\theta^a) = \mu^a_3G^a_{11,\tilde\theta^a,h^a}\tilde\theta^a + \mu^a_4G^a_{12,\tilde\theta^a,h^a}\tilde\theta^a - G^a_{13,\tilde\theta^a,h^a}(\mathbf{u}^a,u^a_3)\tilde\theta^a + F_2^{a(\tilde\theta^a,h^a)},$$

$$\begin{aligned}G^s_{4,\tilde\theta^s,\widetilde S,h^s}(\mathbf{u}^s,u^s_3,\tilde\theta^s) &= \mu^s_3G^s_{11,\tilde\theta^s,\widetilde S,h^s}\tilde\theta^s + \mu^s_4G^s_{12,\tilde\theta^s,\widetilde S,h^s}\tilde\theta^s\\ &\quad -G^s_{13,\tilde\theta^s,\widetilde S,h^s}(\mathbf{u}^s,u^s_3)\tilde\theta^s + F_2^{s(\tilde\theta^s,\widetilde S,h^s)},\end{aligned}$$

$$G^a_{5,\tilde\theta^a,h^a}(\mathbf{u}^a,u^a_3,\tilde q) = \mu^a_5G^a_{11,\tilde\theta^a,h^a}\tilde q + \mu^a_6G^a_{12,\tilde\theta^a,h^a}\tilde q - G_{13,\tilde\theta^a,h^a}(\mathbf{u}^a,u^a_3)\tilde q + F_3^{a(\tilde\theta^a,h^a)},$$

$$\begin{aligned}G^s_{5,\tilde\theta^s,\widetilde S,h^s}(\mathbf{u}^s,u^s_3,\widetilde S) &= \mu^s_5G^s_{11,\tilde\theta^s,\widetilde S,h^s}\widetilde S + \mu^s_6G^s_{12,\tilde\theta^s,\widetilde S,h^s}\widetilde S\\ &\quad -G_{13,\tilde\theta^s,\widetilde S,h^s}(\mathbf{u}^s,u^s_3)\widetilde S + F_3^{s(\tilde\theta^s,\widetilde S,h^s)},\end{aligned}$$

$$\mathbf{G}_{2,\tilde\theta^a,h^a,\tilde\theta^s,\widetilde S,h^s}(\mathbf{u}^a,\mathbf{u}^s) = |\mathbf{u}^a-\delta\mathbf{u}^s|^\alpha(\mathbf{u}^a-\delta\mathbf{u}^s-(\mathbf{u}^a\cdot\mathbf{n}^{a\prime})\mathbf{n}^{a\prime} - \delta(\mathbf{u}^s\cdot\mathbf{n}^{s\prime})\mathbf{n}^{s\prime}).$$

By using the explicit representation of $g_1^{(\tau)} = g_1\circ X_3^\tau$, $\mathscr{V}^{(\tau)}$ and $H^{(\tau)}$ $(\tau = a,s)$ in terms of *y-coordinates*, the last equalities in (2.12) are equivalent to the following equations:

$$\begin{aligned}\frac{\partial h^a}{\partial t} &= L^a_{4,\mathscr{U}^a,h^a}h^a + G^a_{6,\tilde\theta^a,h^a,\tilde\theta^s,\widetilde S,h^s}(\mathscr{U}^a,\mathscr{U}^s),\\ \frac{\partial h^s}{\partial t} &= L^s_{4,\mathscr{U}^s,h^s}h^s + G^s_{6,\tilde\theta^a,h^a,\tilde\theta^s,\widetilde S,h^s}(\mathscr{U}^a,\mathscr{U}^s)\quad\text{in } \mathbf{R}^2_T,\\ (h^a,h^s)|_{t=0} &= \big(h^a_0,h^s_0\big)(y')\quad\text{on } \mathbf{R}^2,\end{aligned}\tag{2.14}$$

where

$$L^a_{4,\mathscr{U}^a,h^a} = \frac{\sigma L}{la(\theta^a_e)(1+|\nabla\Psi^a|^2)}\sum_{i,j=1}^2 c^a_{ij}\frac{\partial^2}{\partial y_i\partial y_j},$$

$$L^s_{4,\mathscr{U}^s,h^s} = \frac{\sigma L}{la(\theta^s_e)(1+|\nabla\Psi^s)|^2)}\sum_{i,j=1}^2 c^s_{ij}\frac{\partial^2}{\partial y_i\partial y_j},$$

$$c^a_{11} = 1+\frac{1}{F^{a2}}\left(E^a\frac{\partial h^a}{\partial y_2}+K^a_{12}\right)^2,\quad c^a_{22} = 1+\frac{1}{F^{a2}}\left(E^a\frac{\partial h^a}{\partial y_1}+K^a_{11}\right)^2,$$

$$c^a_{12} = c^a_{21} = -\frac{1}{F^{a2}}\left(E^a\frac{\partial h^a}{\partial y_1}+K^a_{11}\right)\left(E^a\frac{\partial h^a}{\partial y_2}+K^a_{12}\right),$$

$$c^s_{11} = 1+\frac{1}{g^2}\left(E^s\frac{\partial h^s}{\partial y_2}+K^s_{12}\right)^2,\quad c^s_{22} = 1+\frac{1}{g^2}\Big(E^s\frac{\partial h^s}{\partial y_1}+K^s_{11})^2,$$

$$c^s_{12} = c^s_{21} = -\frac{1}{g^2}\left(E^s\frac{\partial h^s}{\partial y_1}+K^s_{11}\right)\left(E^s\frac{\partial h^s}{\partial y_2}+K^s_{12}\right),$$

$$\begin{aligned}G^a_{6,\tilde\theta^a,h^a,\tilde\theta^s,\widetilde S,h^s}(\mathscr{U}^a,\mathscr{U}^s) &= \frac{1}{E^a}\Bigg(-K^a_2 + \frac{\sigma LF^a}{la(\theta^a_e)(1+|\nabla\Psi^a|^2)}\sum_{i,j=1}^2 c^a_{ij}H^a_{ij}\\ &+\frac{F^a}{la(\theta^a_e)}\big(\widetilde G_{6,\tilde\theta^a,h^a,\tilde\theta^s,\widetilde S,h^s}(\mathscr{U}^a,\mathscr{U}^s) - g_1^{(\tilde\theta^a,h^a)}(1+|\nabla\Psi^a|^2)^{\frac12}|\mathbf{u}^a-\delta\mathbf{u}^s|^\alpha\tilde\theta^a_e\big)\Bigg),\end{aligned}$$

$$G^s_{6,\tilde\theta^a,h^a,\tilde\theta^s,\widetilde S,h^s}(\mathscr U^a,\mathscr U^s)=\frac{1}{E^s}\Bigg(-K^s_2+\frac{\sigma Lg}{la(\theta^s_e)(1+|\nabla\Psi^s|^2)}\sum_{i,j=1}^{2}c^s_{ij}H^s_{ij}$$
$$+\frac{g}{la(\theta^s_e)}\big(\widetilde G_{6,\tilde\theta^a,h^a,\tilde\theta^s,\widetilde S,h^s}(\mathscr U^a,\mathscr U^s)-g_1^{(\tilde\theta^s,\widetilde S,h^s)}(1+|\nabla\Psi^s|^2)^{\frac12}|\mathbf u^a-\delta\mathbf u^s|^\alpha\tilde\theta^s_e\big)\Bigg),$$

$$\widetilde G_{6,\tilde\theta^a,h^a,\tilde\theta^s,\widetilde S,h^s}(\mathscr U^a,\mathscr U^s)=\Bigg\{\mu^a_3\Big(\nabla_{h^a}+a^{33}_a\mathbf F_5^{a(\tilde\theta^a,h^a)}\frac{\partial}{\partial y_3}\Big)\tilde\theta^a\cdot\nabla\Psi^a$$
$$+\mu^a_4a^{33}_a\frac{gp_0}{R\tilde\theta^a}\frac{\partial\tilde\theta^a}{\partial y_3}n^a_3\Bigg\}-\Bigg\{\mu^s_3\Big(\nabla_{h^s}+a^{33}_s\mathbf F_5^{s(\tilde\theta^s,\widetilde S,h^s)}\frac{\partial}{\partial y_3}\Big)\tilde\theta^s\cdot\nabla\Psi^s+\mu^a_4a^{33}_a\tilde\varrho^s g\frac{\partial\tilde\theta^a}{\partial y_3}n^s_3\Bigg\},$$

$$F^a(y',t)=\frac{g}{R\theta^{a**}}|_{y_3=h^a_0(y')},$$

$$E^a(y',t)=\frac{1}{h^a(y',t)}+\frac{p_0-h^a}{p_0-h^a_0}\int_{p_0}^{h^a_0}\frac{(A^a_1+D^a_1)}{\tilde p^a(y,t)\theta^{a**}}\frac{\partial\theta^{a**}}{\partial y_3}\,\mathrm dy_3,$$

$$D^a_1(y,t)=-\frac{\tilde p^a(y,t)}{\theta^{a**}}\int_{p_0}^{y_3}\frac{A^a_1}{\tilde p^a(y,t)}\frac{\partial\theta^{a**}}{\partial y_3}\,\mathrm dy_3,$$

$$\mathbf K^a_1(y',t)=\frac{p_0-h^a}{p_0-h^a_0}\int_{p_0}^{h^a_0}\Bigg(\frac{1}{\tilde p^a(y,t)\theta^{a**}}\Big(\nabla\theta^{a**}+\mathbf B^a_1\frac{\partial\theta^{a**}}{\partial y_3}\Big)+\mathbf L^a_1\Bigg)\mathrm dy_3\equiv\big(K^a_{11},K^a_{12}\big)^{\mathrm T},$$

$$\mathbf L^a_1(y,t)=-\frac{1}{\theta^{a**2}}\frac{\partial\theta^{a**}}{\partial y_3}\int_{p_0}^{y_3}\frac{1}{\tilde p^a(y,t)}\Big(\nabla\theta^{a**}+\mathbf B^a_1\frac{\partial\theta^{a**}}{\partial y_3}\Big)\mathrm dy_3,$$

$$H^a_{ij}=\frac{\partial}{\partial y_i}\Big(\frac{E^a}{F^a}\Big)\frac{\partial h^a}{\partial y_j}+\frac{\partial}{\partial y_i}\Big(\frac{K^a_{1j}}{F^a}\Big)\quad(i,j=1,2),$$

$$K^a_2(y',t)=\frac{p_0-h^a}{p_0-h^a_0}\int_{p_0}^{h^a_0}\Big(\frac{1}{\tilde p^a(y,t)\theta^{a**}}\frac{\partial\theta^{a**}}{\partial t}+L^a_2\Big)\mathrm dy_3,$$

$$L^a_2(y,t)=-\frac{1}{\theta^{a**2}}\frac{\partial\theta^{a**}}{\partial y_3}\int_{p_0}^{y_3}\frac{1}{\tilde p^a(y,t)}\frac{\partial\theta^{a**}}{\partial t}\,\mathrm dy_3,$$

$$E^s(y',t)=\frac{1}{\varrho^{s**}}|_{y_3=h^s_0}+\frac{p_0-h^s}{p_0-h^s_0}\int_{p_0}^{h^s_0}A^s_1\frac{\partial}{\partial y_3}\Big(\frac{1}{\varrho^{s**}}\Big)\mathrm dy_3,$$

$$\mathbf K^s_1(y',t)=g\nabla b+\frac{p_0-h^s}{p_0-h^s_0}\int_{p_0}^{h^s_0}\Big(\nabla+\mathbf B^s_1\frac{\partial}{\partial y_3}\Big)\frac{1}{\varrho^{s**}}\,\mathrm dy_3\equiv\big(K^s_{11},K^s_{12}\big)^{\mathrm T},$$

$$H^s_{ij}=\frac{\partial}{\partial y_i}\Big(\frac{E^s}{g}\Big)\frac{\partial h^s}{\partial y_j}+\frac{\partial}{\partial y_i}\Big(\frac{K^s_{1j}}{g}\Big)\quad(i,j=1,2),$$

$$K^s_2(y',t)=\frac{p_0-h^s}{p_0-h^s_0}\int_{p_0}^{h^s_0}\frac{\partial}{\partial t}\Big(\frac{1}{\varrho^{s**}}\Big)\mathrm dy_3.$$

3. The Main Theorem

Before stating the main theorem, we introduce function spaces (see, for instance, [18, 19]).

Let G be a domain in $\mathbf{R}^n$ $(n = 2, 3)$ and let l be a nonnegative number. By $W_2^l(G)$ we mean the space of functions $u(x)$, $x \in G$, under the norm

$$\|u\|^2_{W_2^l(G)} = \sum_{|\alpha|<l} \|\mathrm{D}^\alpha u\|^2_{L_2(G)} + \|u\|^2_{\dot{W}_2^l(G)},$$

where

$$\|u\|^2_{\dot{W}_2^l(G)} = \begin{cases} \sum\limits_{|\alpha|=l} \|\mathrm{D}^\alpha u\|^2_{L_2(G)} = \sum\limits_{|\alpha|=l} \int\limits_G |\mathrm{D}^\alpha u(x)|^2 \,\mathrm{d}x \quad \text{if } l \text{ is an integer,} \\ \sum\limits_{|\alpha|=[l]} \int\limits_G \int\limits_G \frac{|\mathrm{D}^\alpha u(x) - \mathrm{D}^\alpha u(y)|^2}{|x-y|^{n+2\{l\}}} \,\mathrm{d}x\mathrm{d}y \\ \quad \text{if } l \text{ is a noninteger, } l = [l] + \{l\},\ 0 < \{l\} < 1. \end{cases}$$

Here α is a multi-index. We also define the following function spaces with $m > 1$:

$$\overline{W}_2^m(G) = \Big\{ f(x) \| \|f\|^2_{\overline{W}_2^m(G)} \equiv \sup_{x \in G} |f(x)|^2 + \|f\|^2_{\dot{W}_2^{m-[m]}(G)} + \|\nabla f\|^2_{W_2^{m-1}(G)} < \infty \Big\}.$$

Next, we introduce the anisotropic Sobolev–Slobodetskiĭ spaces

$$W_2^{l,l/2}(G_T) \equiv W_2^{l,0}(G_T) \cap W_2^{0,l/2}(G_T) \quad (G_T \equiv G \times [0,T]),$$

whose norms are defined by

$$\|u\|^2_{W_2^{l,l/2}(G_T)} = \int_0^T \|u(t)\|^2_{W_2^l(G)} \,\mathrm{d}t + \int_G \|u(x)\|^2_{W_2^{l/2}(0,T)} \,\mathrm{d}x$$
$$\equiv \|u\|^2_{W_2^{l,0}(G_T)} + \|u\|^2_{W_2^{0,l/2}(G_T)}.$$

Moreover, we define the function spaces

$$\widetilde{W}_2^{l,l/2}(G_T) = \left\{ f \in W_2^{l,l/2}(G_T) \mid \frac{\partial f}{\partial x_3} \in W_2^{l,l/2}(G_T) \right\}$$

with the norm

$$\|f\|^2_{\widetilde{W}_2^{l,l/2}(G_T)} = \|f\|^2_{W_2^{l,l/2}(G_T)} + \left\| \frac{\partial f}{\partial x_3} \right\|^2_{W_2^{l,l/2}(G_T)},$$

and for $m > 1$,

$$\overline{W}_2^{m,m/2}(G_T) = \big\{ f(x,t) \mid \|f\|^2_{\overline{W}_2^{m,m/2}(G_T)} < \infty \big\},$$

$$\|f\|^2_{\overline{W}_2^{m,m/2}(G_T)} \equiv \sup_{(x,t)\in G_T} |f(x,t)|^2 + \sup_{x \in G} \|f(x)\|^2_{\dot{W}_2^{(m-[m])/2}(0,T)}$$
$$+ \sup_{t \in (0,T)} \|f(t)\|^2_{\dot{W}_2^{m-[m]}(G)} + \|\nabla f\|^2_{W_2^{m-1,(m-1)/2}(G_T)}.$$

The norms of the vector spaces and the product spaces are defined by the standard vector norm and the sum of the norms of each space, respectively.

The following theorem is our main result:

Theorem 3.1. *Let $l \in (1/2, 1)$, and let T be an arbitrary positive number. Assume that*

(i) $\alpha = 2$ *or* $\alpha > 2l + 1$;

(ii) $la(x)$ *is a positive function on* $\mathbf{R}$ *from*

$$C^{2+L}(\mathbf{R}) = \left\{ la(x) \mid \|\!|la|\!\|\equiv \sum_{i=0}^{2} \sup_{x\in\mathbf{R}} \left| \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^i la(x) \right| + \left| \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^2 la(x) \right|^{(L)} < \infty \right\},$$

where $|la|^{(L)}$ *is the Lipschitz coefficient of* la;

(iii) $\mathbf{v}_0^\tau \in W_2^{1+l}(\Omega^\tau)$, $\theta_0^\tau \in \overline{W}_2^{2+l}(\Omega^\tau)$ $(\tau = a, s)$, $q_0 \in \overline{W}_2^{1+l}(\Omega^a)$, $S_0 \in \overline{W}_2^{2+l}(\Omega^s)$, *and* $0 < \underline{\theta}_0 \le \theta_0^\tau(x) < \infty$ $(\tau = a, s)$, $0 < \underline{q}_0 \le q_0 < \infty$, $0 < \underline{S}_0 \le S_0 < \infty$ *hold with positive constants* $\underline{\theta}_0$, $\underline{q}_0$ *and* $\underline{S}_0$;

(iv) θ_e, $S_e \in \overline{W}_2^{3+l,\frac{3+l}{2}}(\mathbf{R}_T^3)$, $\frac{\partial\theta_e}{\partial x_3}$, $\frac{\partial S_e}{\partial x_3} \in W_2^{3+l,\frac{3+l}{2}}(\mathbf{R}_T^3)$, $q_e \in \overline{W}_2^{2+l,\frac{2+l}{2}}(\mathbf{R}_T^3)$, $\frac{\partial q_e}{\partial x_3} \in W_2^{2+l,\frac{2+l}{2}}(\mathbf{R}_T^3)$, *and* $0 < \underline{\theta}_0 \le \theta_e < \infty$, $0 < \underline{q}_0 \le q_e < \infty$, $0 < \underline{S}_0 \le S_e < \infty$ *hold with the same constants* $\underline{\theta}_0$, $\underline{q}_0$, *and* $\underline{S}_0$ *as in* (iii);

(v) θ_b, $S_b \in \overline{W}_2^{\frac{5}{2}+l,\frac{5}{4}+\frac{l}{2}}(\mathbf{R}_T^2)$, *and* $0 < \underline{\theta}_0 \le \theta_b < \infty$, $0 < \underline{S}_0 \le S_b < \infty$ *hold with the same constants* $\underline{\theta}_0$ *and* $\underline{S}_0$ *as in* (iii);

(vi) $\mathbf{v}_c$, $q_c \in W_2^{\frac{1}{2}+l,\frac{1}{4}+\frac{l}{2}}(\mathbf{R}_T^2)$, $\theta_c \in W_2^{\frac{3}{2}+l,\frac{3}{4}+\frac{l}{2}}(\mathbf{R}_T^2)$;

(vii) $d_0 \in W_2^{\frac{3}{2}+l}(\mathbf{R}^2)$, $b \in \overline{W}_2^{\frac{3}{2}+l}(\mathbf{R}^2)$, *and* $H - d_0(x')$, $d_0(x') - b(x') > c_0 > 0$ *holds on* $\mathbf{R}^2$ *for some positive constant* c_0;

(viii) F_2^τ, $F_3^\tau \in \widetilde{W}_2^{1+l,\frac{1+l}{2}}(\mathbf{R}_T^3)$, $\mathbf{F}_1^\tau \in \widetilde{W}_2^{l,\frac{l}{2}}(\mathbf{R}_T^3)$ $(\tau = a, s)$, *and their derivatives with respect to* x_3 *are Hölder continuous with exponent* $\beta \in (l/2, 1)$ *with respect to* x_3;

(ix) $g_1 \in \widetilde{W}_2^{2+l,\frac{2+l}{2}}(\mathbf{R}_T^3)$.

Moreover, the compatibility conditions of order 0 *for* $(\mathbf{u}^\tau, u_3^\tau, \tilde{q}, h^\tau)$ *and of order* 1 *for* $\tilde{\theta}^\tau$ *and* $\widetilde{S}$ *are satisfied for* $\tau = a$, s.

Then there exists $T^* \in (0, T]$ *such that the problem* (2.12)–(2.14) *has a unique solution*

$$\begin{aligned}
\left(\mathbf{u}^a, u_3^a, \tilde{\theta}^a, \tilde{q}, h^a\right) \in Z^a(T^*) \equiv{}& W_2^{2+l,\frac{2+l}{2}}\left(\widetilde{\Omega}_{T^*}^a\right) \times \widetilde{W}_2^{1+l,\frac{1+l}{2}}\left(\widetilde{\Omega}_{T^*}^a\right) \times W_2^{3+l,\frac{3+l}{2}}\left(\widetilde{\Omega}_{T^*}^a\right)\\
&\times W_2^{2+l,\frac{2+l}{2}}\left(\widetilde{\Omega}_{T^*}^a\right) \times W_2^{\frac{5}{2}+l,\frac{5}{4}+\frac{l}{2}}\left(\mathbf{R}_{T^*}^2\right),\\
\left(\mathbf{u}^s, u_3^s, \tilde{\theta}^s, \widetilde{S}, h^s\right) \in Z^s(T^*) \equiv{}& W_2^{2+l,\frac{2+l}{2}}\left(\widetilde{\Omega}_{T^*}^s\right) \times \widetilde{W}_2^{1+l,\frac{1+l}{2}}\left(\widetilde{\Omega}_{T^*}^s\right) \times W_2^{3+l,\frac{3+l}{2}}\left(\widetilde{\Omega}_{T^*}^s\right)\\
&\times W_2^{3+l,\frac{3+l}{2}}\left(\widetilde{\Omega}_{T^*}^s\right) \times W_2^{\frac{5}{2}+l,\frac{5}{4}+\frac{l}{2}}\left(\mathbf{R}_{T^*}^2\right)
\end{aligned}$$

satisfying

$$0 < \tilde{\theta}^\tau < \infty \quad \text{on } \widetilde{\Omega}^\tau \times [0, T^*] \quad (\tau = a, s),$$
$$0 < \tilde{q} < \infty \quad \text{on } \widetilde{\Omega}^a \times [0, T^*], \quad 0 < \widetilde{S} < \infty \quad \text{on } \widetilde{\Omega}^s \times [0, T^*].$$

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